1. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long- term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.
   1. Clearly define the decision variables
   2. What is the objective function?
   3. What are the constraints?
   4. Write down the full mathematical formulation for this LP problem.

**Solution:**

Assume that the company has to produce the X1 numbers of collegiate and X2 numbers of mini backpacks.

Since each collegiate generates $32 per unit profit and each mini backpack generates $24 per unit profit, so the total profit will be denoted by Z.

As per the question, each collegiate requires 3 square feet of nylon fabric and each mini backpack requires 2 square feet of nylon fabric, so the total nylon fabric required will be

Also, the sales forecast indicates at most 1000 collegiates and 1200 mini backpacks can be sold per week, therefore

Total labor time required to produce X1 collegiate and X2 mini backpacks is (45X1 + 40X2) minutes.

Available labor time is 35\*40\*60 = 84000 minutes

Therefore,

Hence, the mathematical problem should be defined as following:

The decision variables are:

X1 = number of collegiates

X2 = number of mini backpacks

Maximize

Subject to

and

1. The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant’s excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

a. Define the decision variables

b. Formulate a linear programming model for this problem.

**Solution:**

Assume that the plant 1 produces X1, Y1 & Z1 units of size large, medium and small respectively. And plant 2 produces X2, Y2 & Z2 units of size large, medium and small respectively. Similarly, plant 3 produces X3, Y3 & Z3 units of size large, medium and small respectively. So, the total profit will be calculated as:

3

Where, Z denotes the total net profit per day.

Now, plant 1 can produce 750 units per day. And plant 2 & 3 can produce 900 and 450 units respectively. Therefore,

Also, Plant 1, 2 & 3 have 13000, 1200 & 5000 square feet of space available respectively for in-process storage of a day’s production. And each unit of large, medium and small size products require 20, 15 & 12 square feet respectively. Therefore,

And the sales forecast indicates that 900, 1200 & 750 units of large, medium and small sizes respectively are sold per day. So,

The plant should use the same percentage of their excess capacity to produce the new products. So,

(This will be redundant, therefore can be removed)

Hence, the Linear programming model should be defined as:

The decision variables are:

X1 = number of large units produced per day at Plant 1

Y1 = number of medium units produced per day at Plant 1

Z1 = number of small units produced per day at Plant 1

X2 = number of large units produced per day at Plant 2

Y2 = number of medium units produced per day at Plant 2

Z2 = number of small units produced per day at Plant 2

X3 = number of large units produced per day at Plant 3

Y3 = number of medium units produced per day at Plant 3

Z3 = number of small units produced per day at Plant 3

Maximize

subject to

And